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Review on Dynamic Responses of Vibrating Systems with Uncertain Parameters

Imran Ahemad Khan^{*1}, G. K. Awari²

^{*1}Faculty Mechanical Department, Priyadarshini College of Engineering,
Nagpur, India

²Principal, Tulsiramji Gaikwad-Patil College of Engineering and Technology, Nagpur, India
iak20041978@rediffmail.com

Abstract

This paper is based on the study of dynamic responses of various vibrating systems at multiple frequencies with uncertain parameters. The concept of uncertainty plays an important role in the design of engineering systems. Uncertainty can be in the form of no homogeneous material, frictional changes in damping and variation in spring stiffness during operation. Dynamic response of a mechanical system may be affected adversely due to existence of minor uncertainty. Hence it becomes important to investigate its effects on mechanical system for different frequency domain. Large and varied amount of research has been carried out in developing the techniques which predict the dynamic responses of structures in frequency and time domains. However studies related with the dynamic responses of structures with uncertainty are limited. The aim of this paper is to study, analyze, compare and report, available investigation that emphasizes uncertainty of at least one type i.e. either related with spring or mass or damper.

Keywords: Vibration Analysis, Modal Analysis, Uncertain parameters, FEA and SEA

Introduction

For to calculating the natural frequencies and mode shapes of a structure Modal analysis method is used. This method determined the dynamic response of complicated structural dynamic problems. In general, applications of modal analysis today cover a broad range of objectives identification and evaluation of vibration phenomena, validation, structural integrity assessment, structural modification, and damage detection. It applies on mechanical systems, transportation systems, civil engineering structures and anything that is subject to dynamic motions or vibration.

In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of given set of design parameters. The study of mathematical models which involve physical and geometric parameters such as mass density ρ , elastic modulus E , Poisson's ratio ν , lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes. In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an

important role in investigation of various engineering problems.

The frequency spectrum where simulation methods can be utilized for vibration analysis has three domains: low, mid, and high frequency.

The low-frequency domain is defined as the frequency range where all components are short with respect to a wavelength. Short members respond with low modal density, and resonant effects are dominant. In this domain no uncertainties with respect to the natural frequencies or the dimensions of the short members are considered. For simulating low-frequency vibrations conventional Finite Element Analysis (FEA) is a practical numerical approach.

The high-frequency domain is defined as the frequency range where all component members of a system are long with respect to a wavelength. For simulations at high frequencies Statistical energy analysis (SEA) and energy finite element analysis (EFEA) can be used.

The mid-frequency domain is defined as the frequency range where some of the components of a system are long and other members are short. Finite Element Analysis is used for short component and Statistical energy analysis for long component. For many structures there exists a region between the two techniques, often referred to as the mid frequency region,

where neither technique can be accurately applied. Therefore it becomes important study which method will be helpful to analysis the mid-frequency domain problems.

In last two decades, a large and varied amount of research has been dedicated to developing techniques which predict the dynamic responses of structures with uncertainty. This literature review aims to cover the majority of techniques in this field.

I. Methods to Predict Dynamic Response of Mechanical System with Uncertain Parameters

In this review paper year wise Methods to Predict Dynamic Response of Mechanical System with Uncertain Parameters were discussed.

In 1995 Z. Fang [1] suggested a combination of the transfer matrix method and a first order, second moment approach is used to perform dynamic analyses of structures with uncertain parameters. This method determines vibrational characteristics of structures with uncertain parameters and also the first and second statistical moments of these parameters as eigenvalues and the corresponding vibrational mode shapes of the system using $E(X_i) = 0 \ (I = 1, 2, \dots, r)$, they will be able to obtain the formulae which are used to calculate the mean values and variances of eigenvalues $\lambda_{(s)}$ and the corresponding vibrational mode shapes $\varphi_{(s)}$, of the system, which is composed of n segment. The method involves very easy calculation and uses to determine the dynamic analysis of a beam system with uncertain parameters as shown in Fig.1 and also structural engineering problems exhibit only minor deviation from the mean.

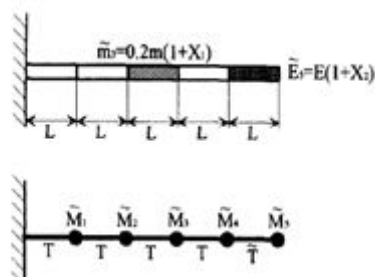


Fig1: Cantilever Beam System

In 2003 Su Huan Chen and Jie Wu, Yu Dong Chen [2] presents an interval optimization method to solve the uncertain problems of the vibration systems with multi-degrees of freedom, where the structural characteristics are assumed to be expressed as interval parameters. This method is form by combining interval extension of function with the first-order Taylor

expansions of the functions. Using the interval optimization method, more information for the optimal structures can be obtained, such as how the optimization results change if the uncertainties of structural parameters are imposed on the structures. This method is used for a torsional vibration system and a frame structure.

In this they will apply the interval optimization method to the linear torsional vibration system with n degrees of freedom as shown in Fig.2

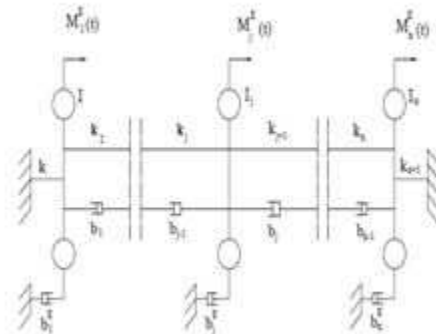


Fig. 2. Linear torsional vibration system with n degrees of freedom.

The vibration equation of the linear torsional vibration system as shown in Fig. 2 is

$$M(I_i)\ddot{q}(t) + B(b_i, b_i^E)\dot{q}(t) + K(K_i)q(t) = F_c \cos \alpha t + F_s \sin \alpha t \quad (1)$$

where $F_c = \{M_{ic}/T$, $F_s = \{M_{is}/T$, $M(I_i)$ is the mass matrix, $B(b_i; b_i^E)$ the damping matrix, and $K(K_i)$ the stiffness matrix. The solution of the equation $q(X) = G(X)(F_c - iF_s) = \{q_i(X)\} \quad (2)$

In 2004 David Moens and Dirk Vandepitte [3] review Interval and Fuzzy Finite Element Analysis the emerging non-probabilistic approaches for uncertainty treatment in finite element analysis. In the first part, the applicability of the non-probabilistic concepts for numerical uncertainty analysis is discussed from a theoretical viewpoint. The second part of the paper focuses on numerical aspects of the interval finite element method. In this paper it shows that non-probabilistic analysis is more precise than probabilistic analysis approach. The interval and fuzzy approach becomes useful in the presence of uncertain quantities that require subjective information in order to be described numerically and tool for an analyst who wants to study the effect of uncertainties of which he has expert knowledge or which he wants to optimize in the design. This makes the non-probabilistic approaches most valuable in early design stages, whereas the probabilistic approach remains indispensable in later stages. This paper further shows that the popular interval arithmetic

implementation of the IFE procedure is extremely vulnerable to conservatism, especially during the interval system matrices assembly phase.

In 2005 Zhiping Qiu and Xiaojun Wang [4] presents a new algorithm of the parameter perturbation method and interval mathematics. It was determining the range on the dynamic response of structures with uncertain-but-bounded parameters. It need less prior knowledge on uncertain parameters than the probabilistic approach. On the basis of mathematical proofs and numerical simulations comparison is done between the parameter perturbation method and the probabilistic approach. The numerical results are in agreement with the mathematical proofs. The response range given by the parameter perturbation method encloses that obtained by the probabilistic approach. The results indicate the strangeness of the approach.

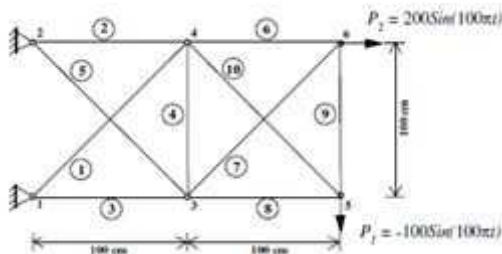


Fig 3. A 10-bar two-dimensional truss.

In order to illustrate the effectiveness of the presented parameter perturbation method for the dynamic responses of the structures with uncertainty, we apply it to a two-dimensional truss with 6 nodes and 10 elements as shown in Fig. 3. Young’s modulus of the element material is $E = 2.0 \times 10^{11} \text{N/m}^2$ and the element mass density is $\rho = 7800.0 \text{ kg/m}^3$. The dimension and boundary conditions of the truss are indicated in Fig. 3. Now it is assumed that there are two harmonic sinusoidal excitations $P_1 = -100\text{Sin}(100\pi t) \text{ N}$ and $P_2 = 200\text{Sin}(100\pi t) \text{ N}$ acting on the node 5 in the vertical direction and the node 6 in the horizontal direction respectively, with the initial conditions $x(t) = 0$ and $\dot{x}(t) = 0$. Due to the manufacture errors or measurement errors, the cross-sectional areas of elements exhibit some uncertainties, and are considered to be uncertain but-bounded parameters. Their intervals are taken as

$$A_i^l = [A_i^c - k\beta A_i^c, A_i^c + k\beta A_i^c], i = 1, 2, \dots, 10 \quad (3)$$

Where $A_i^c = 1.0 \times 10^{-4} \text{ m}^2$, β is a variable coefficient and k is a positive integer. Here β is taken as 0.005, and k is taken as 10. In order to compare with probabilistic approach, these uncertain parameters also are assumed to be random variables with mean value $\mu_{A_i} = A_i^c$ and standard variance $\sigma_{A_i} = \beta A_i^c$. In the following, the

displacement responses of the node 5 in the vertical direction are calculated using the presented parameter perturbation method in comparison with the probabilistic approach.

In 2005 G. Manson[5] the two methods complex interval analysis and complex affine analysis are compare for the effect of parametric uncertainty upon the response of mechanical systems. He found complex affine analysis performed significantly better than its complex interval analysis in following manner that it returns significantly tighter bounds on the true solution set, sensitivity information is returned by affine arithmetic relating to each independent source of uncertainty and the measure of usefulness of the prediction via examination of approximation errors which permit the calculation of inner and outer bounds between which the true solution set boundaries must lie. Complex affine analysis is used where interval analysis has been disregarded as a tool for dealing with uncertainty due its conservative nature and interval analysis is presently used but dependency issues, perhaps weak, exist.

In 2006 Wei Gao [6] in this paper the natural frequency and mode shape analysis of truss structures with uncertain parameters are determine by two methods called random factor method and interval factor method . In this paper he developed expressions of the mean value and standard deviation, lower and upper bounds of the natural frequency and mode shape. The example is that the dynamic characteristics analytical results of the structure (as show in Fig. 4.) with uncertainty can be obtained appropriately by Random Factor Method and Interval Factor Method.

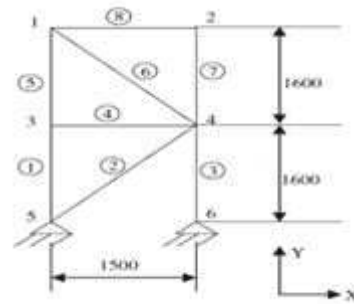


Fig. 4. An 8-bar planar truss structure (unit: mm).

The natural frequency can be found by given formula

$$\Delta\omega = \frac{(\bar{\omega} - \omega)}{2} = \frac{1}{2} \left[\left(\frac{1 + \Delta E_f}{(1 - \Delta \rho_f)(1 - \Delta L_f)(1 - \Delta L_f)} \right)^{1/2} - \left(\frac{1 - \Delta E_f}{(1 + \Delta \rho_f)(1 + \Delta L_f)(1 + \Delta L_f)} \right)^{1/2} \right] \omega^* \quad (4)$$

The midpoint value and maximum width of mode shape can be obtained by given formula

$$\Delta\phi_j = \frac{(\bar{\phi}_j - \phi_j)}{2} = \frac{1}{2} \left\{ \left[\frac{1}{\sqrt{(1-\Delta\sigma_f)(1-\Delta\Delta_f)(1-\Delta\mathcal{L}_f)}} \right]^{\sqrt{2}} - \left[\frac{1}{\sqrt{(1+\Delta\sigma_f)(1+\Delta\Delta_f)(1+\Delta\mathcal{L}_f)}} \right]^{\sqrt{2}} \right\} \phi_j^*$$

(5)

In 2006 JianBing Chen and Jie Li [7] they developed new concept of probability density evolution method, which is capable of capturing the instantaneous probability density function and its evolution of the responses of nonlinear stochastic structures. In this concept, a virtual stochastic process associated to the extreme value of the stochastic response of structures. The extreme value distribution could then be evaluated by dealing with a virtual stochastic dynamical system. Two numerical examples of the extreme value distribution of random sampling and the extreme value distribution and dynamic reliability of nonlinear stochastic structures are studied. In this investigation they found the error of the proposed method mainly lies in the discretization in the numerical procedures, including the error in selecting the representative points from the domain and the error in the finite difference method. Further improvement is needed in the proposed methods.

In 2007 JongSok Sim, Zhiping Qiu and Xiaojun Wang [8] they proposed interval analysis with uncertain-but-bounded parameters method. For more reliable analysis knowledge of bounds of the ranges of structural characteristic needed. In many cases their probable range of values i.e. upper and lower bounds, can be provided from practical experience and engineering knowledge. By using this concept they investigate the method of computing upper and lower bounds of parameters such as, natural frequencies, modal shapes, and FRFs. They solved tower structure problem, and the results illustrate that the proposed method is effective. A comparison is done between the modal interval method with the results of Monte Carlo simulation serves to validate the solutions and to identify the bounded ranges of parameters as shown in following fig.5(a)and fig.5(b).

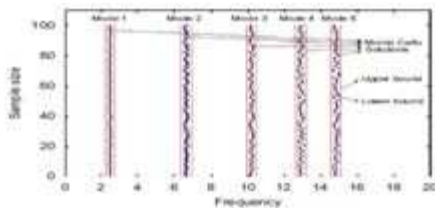


Fig.5(a). Evaluation of bounded results compared with the modal interval analysis with Monte Carlo method

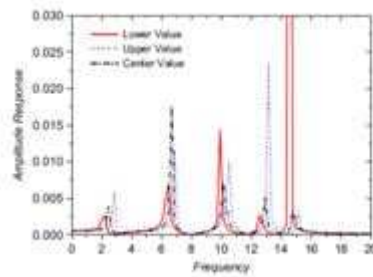


Fig.5(b). Bounded natural frequency (rad/s) of the system

In 2008 Xiao-Ming Zhang and Han Ding [9] developed the convex model method to solve uncertainty problems. Uncertainty parameters are to model as a random vector and to solve it probability density function used. But in this probability distribution is difficult to obtain. So now they use nonprobabilistic convex model to deal with the uncertain parameters in which there is no need for probability density functions. This method gives more details about how the optimization results change if the uncertainties of parameters are imposed on the system. In present work torsional vibration system (as shown in fig.6.) was taken and solved on the basis of first order Taylor expansion because uncertain parameters are small. For the large uncertain parameters second order Taylor expansion will be used.

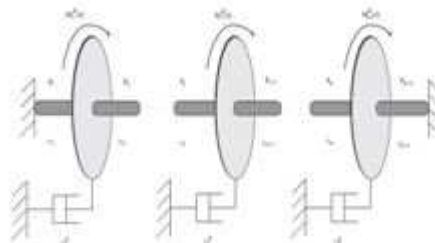


Fig. 6. Linear torsional vibration system with n degrees of freedom.

The vibration equation of the linear torsional vibration system as shown in Fig. 6 is

$$M(I_i)\ddot{x}(t) + C(c_i, c_i^E)\dot{x}(t) + K(k_i)x(t) = F_c \cos \omega t + F_s \sin \omega t \quad (6)$$

In 2009 Nicole J Kessissoglou and Geoff I Lucas [10] for finding the natural frequencies of a dynamic system by what extent Gaussian orthogonal ensemble

statistics applicable are studied. The effect of a range of uncertainties on the modal statistics of mass and /or spring loaded plates and plates coupled by springs are numerically characterized. A non-dimensional statistical overlap factor is randomly varied in an individual natural

frequency from its mean value and is important to predict the frequency beyond which the resonant behavior of individual modes no longer dominates the response statistics. Using a first-order perturbation analysis, an approximate expression for the statistical overlap factor has been developed for the randomised plates, to estimate the modal range for the occurrence of Gaussian orthogonal ensemble statistics.

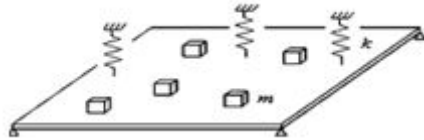


Fig. 7. A simply supported plate with randomly located point masses and springs to ground.

$$\ddot{q}_{1,pq} + k\phi_{1,pq}(x_1) \sum_{mn} q_{1,mn} \phi_{1,mn}(x_1) - k\phi_{1,pq}(x_1) \sum_{jk} q_{2,jk} \phi_{2,jk}(x_1) = 0 \quad (7)$$

$$\ddot{q}_{2,pq} + k\phi_{2,pq}(x_2) \sum_{mn} q_{2,mn} \phi_{2,mn}(x_2) - k\phi_{2,pq}(x_2) \sum_{jk} q_{1,jk} \phi_{1,jk}(x_1) + \omega_{pq}^2 q_{2,pq} = 0 \quad (8)$$

Above two equations can easily be expanded to account for N number of randomly located springs, as shown in Fig. 7. The natural frequencies of the spring-coupled plates were obtained by eigenvalue analysis using MatLab

In 2010 A. Cicirello and R. S. Langley [11] they used hybrid FE –SEA methods and combined parametric and non-parametric modeling uncertainties. A two degree of freedom

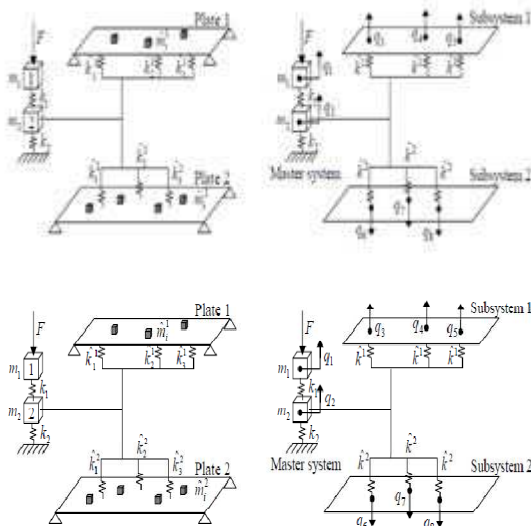


Figure 8 : Structure under study (benchmark and hybrid model)

mass-spring system is analyze and tested to the proposed approaches showing very promising results for future applications to more complex problems.

$$\ddot{q}_r + \sum_{i=1}^{N_i} m_i q_r \phi_r(x_i, y_i) \phi_s(x_i, y_i) + \sum_{j=1}^{N_j} k_j q_r \phi_r(x_i, y_i) \phi_s(x_i, y_i) + \omega_r^2 q_r = 0 \quad (9)$$

The equation of motion of the coupled system can be thus written in the form

$$M\ddot{q} + Kq = F \quad (10)$$

Where M and K are the mass and stiffness matrix, respectively, and F is the force vector. The random distributed masses and springs attached to each plates produce extra terms in the diagonal elements of the mass, M_{Rp} , and stiffness matrix, K_{Rp} , of the plates ($p=1,2$). In addition, the coupling between the two plates by means of the 2DoF leads to off-diagonals term in the stiffness matrix K_{cp} .

In 2011 Xu Wang [12] a hybrid of the deterministic analysis and statistical energy analysis approaches has been proposed in this paper. For the frequency range of 0–600 Hz the three approaches of the SEA, deterministic analysis and the new deterministic-statistical analysis are applied to validate case of plate–acoustic system analysis. The deterministic-statistical analysis is most suitable in the mid-frequency range, e.g., 100–600 Hz for the plate–acoustic system in consideration of reasonable computational accuracy, information and speed. The new deterministic-statistical analysis approach is applicable to other vibro-acoustic system in the same way as to the plate–acoustic system. The new deterministic-statistical analysis approach developed in this paper has largely simplified.

In 2011 Giuseppe Quaranta [13] presents the finite element analysis method by taking into account probability density functions whose parameters are affected by fuzziness. It is shown that the proposed methodology is a general and versatile tool for finite element analyses because it is able to consider, both, probabilistic and non-probabilistic sources of uncertainties, such as randomness, vagueness, ambiguity and imprecision. The main drawback is that, the presence of fuzzy parameters requires a non-negligible additional computational effort. Further studies are needed to improve the proposed method.

In 2012 A.L. Morales, J.A.Rongong and N.D.Sims [14] used a fuzzy design method in the finite element procedure to simulate and analyze active vibration control of structures subjected to uncertain parameters. The basic purpose of this method is to study effect of uncertainty propagation on vibration control

system and avoid the use of expensive probabilistic methods or complex robust control techniques. This method helps the engineer to take the decision of worst and the best case of the design control due to uncertainty propagation.

In 2012 Baizhan Xia and Dejie Yu [15] two interval analysis methods are used i.e. IPFEM and MIPFEM, for the uncertain acoustic field prediction with uncertain-but-bounded parameters. The first-order Taylor series is used to solve interval matrices and vectors. For 2D and 3D acoustic cavity the accuracy of MIPFEM is higher than the accuracy of IPFEM. But the additional computational cost of MIPFEM is the disadvantage for the acoustic problem with a large number of uncertain-but-bounded parameters.

I. Mid Frequency Analysis

Now in the following literature survey work is done on the problems falling in the mid frequency domain. As for low and high frequency domain there are precise methods which can find natural frequency accurately but for mid frequency region no accurate method is their so it become necessary to find it. Some of work is done in this area in which two methods are club and taking advantage of each (hybrid) and finding vibrational characteristics of structure .Some of them are listed below.

In 2000 Xi Zhao and Nickolas Vlahopoulos [16] investigate hybrid finite element method for mid frequency vibrations it contains some long wavelength and short wave length. They used energy finite element method for long wavelength and finite element for short wavelength. A system of interface equations is developed for the joints between long and short members. The interface equations are solved simultaneously through an iterative but computationally efficient process.

Following is the interface equation developed for the joints between long and short members:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & ik_w & k_w & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & ik_w \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ S_{11} & S_{12} & S_{13} & S_{14} & i(EI)_w k_w^2 & -(EI)_w k_w^2 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & S_{24} & -(EI)_w k_w^2 & (EI)_w k_w^2 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & S_{34} & 0 & 0 & i(EI)_w k_w^2 & -(EI)_w k_w^2 \\ S_{41} & S_{42} & S_{43} & S_{44} & 0 & 0 & (EI)_w k_w^2 & -(EI)_w k_w^2 \end{bmatrix} \begin{bmatrix} u_w \\ du_w \\ dv_w \\ dw_w \\ p_w \\ A_w \\ B_w \end{bmatrix} = \begin{bmatrix} 1 \\ ik_w \\ 0 \\ 0 \\ 0 \\ (EI)_w k_w^2 \\ 0 \end{bmatrix} A_w + \begin{bmatrix} 0 \\ 0 \\ 1 \\ ik_w \\ 0 \\ i(EI)_w k_w^2 \\ -(EI)_w k_w^2 \end{bmatrix} C_w + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_{Rw} \\ F_{Rw} \\ M_{Rw} \end{bmatrix}$$

In 2002 Abhijit Sarkar and Roger Ghanem [17] used Stochastic Finite Element Approach for the formulation detailed particularly useful for the mid-frequency vibration analysis of the large scale structural systems in order to investigate the effect of parameter uncertainty on the vibrational responses. They combine stochastic finite element approach based on the orthogonal expansions and projections with the extended energy operator approach to solve such problems.

In 2003 G. Stefanou and M. Papadrakakis [18] in this study the triangular composite facet shell element TRIC

(as shown in fig 9) is presented for the case of combined uncertain material and geometric properties.

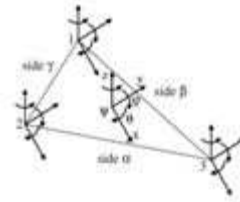


Fig.9. The multi-layered triangular shell element TRIC

The stochastic finite element analysis of shell structures is performed using the spectral representation method for the description of the random fields in conjunction with Monte Carlo simulation (MCS) for the computation of the response variability.

In 2005 P.J. Shorter and R.S. Langley [19] find Hybrid Finite Element-Statistical Energy Analysis Method. They introduced general method for predicting the ensemble average steady-state response of complex vibro-acoustic systems that contain subsystems with uncertain properties. The method combines deterministic and statistical techniques to produce a non-iterative hybrid method that incorporates equations of dynamic equilibrium and power balance .The method is derived explicitly without reference to statistical energy analysis (SEA). The method is therefore suitable for the mid frequency problem. This method is limited in use due to the computational load of the FEM models.

In 2005 Bas van Hal, Wim Desmet, Dirk Vandepitte, Paul Sas [20] they used Hybrid Finite Element - Wave Based Method. This hybrid method is used to predict response of mid frequency range. As FEM is widely accepted for the steady-state dynamic response analysis of acoustic systems but its application is practically limited to the low-frequency range. The wave based method has better convergence properties than the FEM and therefore allows accurate predictions at higher frequencies. So they proposed combining these two methods, the parts of the problem domain with a complex geometry are modeled using the FEM, while the remaining parts are described with a wave based model. The final hybrid FE-WB sub models in equations is as given below

$$\begin{bmatrix} Z^E & C^{EF} & O \\ C^{FE} & O & C^{FW} \\ O & C^{WF} & (A^W + C^{WW}) \end{bmatrix} \begin{Bmatrix} p^E \\ v^F \\ p^W \end{Bmatrix} = \begin{Bmatrix} f^E \\ c^{FW} \\ b^W + c^{WW} \end{Bmatrix}$$

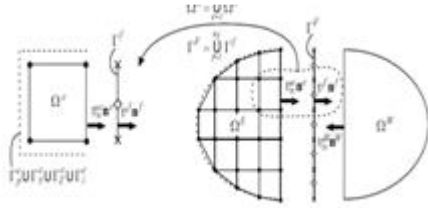


Figure 10: Coupled FE-WB approach

In 2006 Sang Bum Hong, Aimin Wang and Nickolas Vlahopoulos [21] a new approach is introduced in hybrid finite element analysis. The hybrid FEA method combines the conventional FEA method with energy FEA (EFEA) for analysis of systems that contain both flexible and stiff members. A formulation for analyzing flexible plates spot-welded to stiff beams when the excitation is applied on the stiff members is developed.

$$Q_{in} = \langle F \cdot v \rangle = \langle v^2 \rangle / G(x_0, y_0, t) \tag{11}$$

where F represents the force exerted on the flexible member from the stiff member at the spot-welded attachment point (x_0, y_0) , v is the corresponding transverse velocity of the flexible member at the attachment point (x_0, y_0) which is equal to the transverse velocity of the stiff member at the attachment point (x_0, y_0) based on the displacement compatibility at that point, and $\langle \rangle$ represents the time average of the quantity inside the bracket. The power flow at each spot-welded attachment point (x_0, y_0) defines the excitation at the corresponding locations of the EFEA model of the flexible members.

Conventional FEA models are employed for modeling the behavior of the stiff members in a system and an EFEA analysis is performed in order to determine the amount of vibrational energy in the flexible members.

In 2007 Vincent Cotonia and Phil Shorter [22] in this study they combine finite element and statistical energy analysis methods. For low frequency FE is used and for high frequency SEA method is used. But for mid frequency no as such definite method is available. So combining the two methods to get their effects as FE investigates the behavior of long wavelength and SEA investigates the behavior of short wavelength.

$$\omega(\eta_j + \eta_{d_j})E_j + \sum_k \omega \eta_{j_k} \eta_j (E_j / n_j - E_k / n_k) = p_{in}^{ext} \tag{12}$$

$$S_{qq} = D_{tot}^{-1} \left[S_{ff} + \sum_k \left(\frac{4E_k}{\omega m_k} \right) \text{Im} \{ D_{dir}^{(k)} \} \right] D_{tot}^{-1*T} \tag{13}$$

Above two equations are the main equations of the hybrid method. The method yields the ensemble average response of the system where the uncertainty is confined in the SEA subsystems.

In 2007 A. Pratellesi, M. Viktorovitch, N. Baldanzini and M. Pierini [23] they present Hybrid Finite Elements And Smooth Integral Formulation method for investigate behavior of structures in the mid-frequency range. The hybrid formulation is based on the coupling of two different formulations, the finite elements for the low-frequency behaving subparts and a probabilistic formulation, the smooth integral formulation, applied to the high-frequency subsystems.

In 2011 Bert Van Genechten, Dirk

Vandepitte and Wim Desmet [24] this paper deals with hybrid finite element and Wave based modeling method. This method is applied for steady-state coupled structural-acoustic problems.

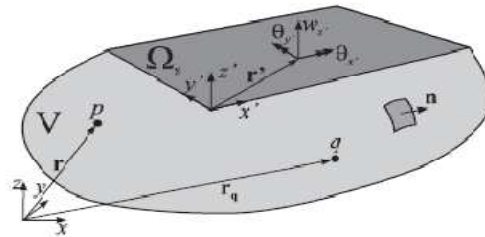


Fig. 11. A 3D coupled structural-acoustic problem

In this approach combination of the advantages of both techniques are used. FEM is used for the structural part and Wave based model for the acoustic cavity.

In 2011 Karel Vergote, Bert Van Genechten, Dirk Vandepitte, and Wim Desmet [25] this paper is based on Hybrid Wave Based Modeling and Statistical Energy Analysis Technique for steady-state dynamic analysis of vibro-acoustic problems in the mid-frequency range. The concept is as shown in the given below fig.12.

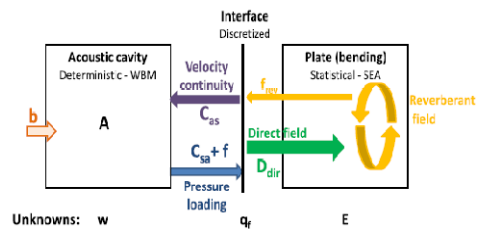


Fig. 12. Concept of the vibro-acoustic hybrid WBM-SEA.

Paper compares the FEM and WBM method with each other. FEM could not effectively calculate the small system matrices and the fast convergence of the method as WBM. The hybrid WBM-SEA predicts the average response of a combined vibro-acoustic

deterministic statistic system, and this for several damping values.

Conclusion

As per the above review of all the research papers, it is come to know that there are so many methods to determine natural frequency of system with and without uncertain parameters. They all are restricted to the frequency domain i.e. low, medium, high frequency. As some of them such as FEM are very accurate for low frequency domain and some are good for high frequency domain such as SEA. But as such no conventional method is confident for mid frequency domain so hybrid methods are evolved to find the response of structure.

The effect of uncertain parameters in low frequency region is very less but in high frequency region it is very dominating. So it becomes necessary to study effect of uncertain parameters on vibrationally excited structures.

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